

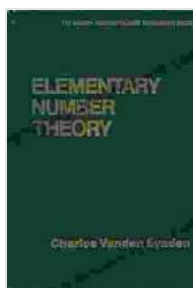
Elementary Number Theory Charles Vanden Eynden

Elementary number theory is a branch of mathematics that deals with the properties of positive integers. It is concerned with questions such as:

- What is the greatest common factor of two numbers?
- What is the least common multiple of two numbers?
- Are two numbers relatively prime?
- What is the prime factorization of a number?
- How many positive integers less than a given number are relatively prime to a given number?

Elementary number theory has many applications in other areas of mathematics, such as abstract algebra, combinatorics, and analysis. It is also used in computer science, cryptography, and statistics.

In this article, we will provide a comprehensive overview of elementary number theory. We will cover the basics of the subject, as well as some more advanced topics. We will also provide many examples and exercises to help you learn the material.



Elementary Number Theory by Charles Vanden Eynden

★★★★☆ 4.3 out of 5

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Divisibility

One of the most basic concepts in number theory is divisibility. A number (a) is divisible by a number (b) if there exists a number (c) such that $(a = bc)$. We write this as $(b \mid a)$.

For example, 12 is divisible by 3 because $12 = 3 * 4$. We write this as $3 \mid 12$.

There are many properties of divisibility that can be used to solve problems. For example, if $(a \mid b)$ and $(b \mid c)$, then $(a \mid c)$. This property is known as the transitive property of divisibility.

Another important property of divisibility is the Euclidean algorithm. The Euclidean algorithm can be used to find the greatest common factor (GCF) of two numbers. The GCF of two numbers is the largest number that divides both numbers.

To find the GCF of two numbers, use the following steps:

1. Divide the larger number by the smaller number.
2. If the remainder is 0, then the smaller number is the GCF.
3. Otherwise, divide the smaller number by the remainder.
4. Repeat steps 2 and 3 until the remainder is 0.
5. The last non-zero remainder is the GCF.

For example, to find the GCF of 12 and 18, we use the following steps:

1. $18 \div 12 = 1$ remainder 6
2. $12 \div 6 = 2$ remainder 0

The last non-zero remainder is 6, so the GCF of 12 and 18 is 6.

Prime Numbers

A prime number is a positive integer greater than 1 that has exactly two factors, 1 and itself. For example, 2, 3, 5, 7, and 11 are all prime numbers.

The prime numbers are the building blocks of the natural numbers. Every positive integer can be written as a product of prime numbers. This property is known as the fundamental theorem of arithmetic.

There are many important properties of prime numbers. For example, there are infinitely many prime numbers. This was proven by Euclid in the 3rd century BC.

Another important property of prime numbers is that they are distributed randomly. This means that there is no way to predict whether or not a given number is prime.

Prime numbers have many applications in mathematics, computer science, and cryptography. For example, prime numbers are used in the RSA encryption algorithm, which is one of the most widely used encryption algorithms in the world.

Congruence

Congruence is a relation between two integers that is used to determine whether or not they have the same remainder when divided by a given number. Two integers (a) and (b) are congruent modulo (m) if (a - b) is divisible by (m). We write this as $(a \equiv b \pmod{m})$.

For example, 12 is congruent to 18 modulo 6 because $12 - 18 = -6$, which is divisible by 6. We write this as $12 \equiv 18 \pmod{6}$.

Congruence has many applications in number theory. For example, congruence can be used to solve Diophantine equations. Diophantine equations are equations that have integer solutions.

Congruence can also be used to study the properties of groups. A group is a set of elements that has an operation that combines any two elements to produce a third element. Groups are used in many areas of mathematics, such as algebra, geometry, and topology.

Applications of Elementary Number Theory

Elementary number theory has many applications in other areas of mathematics, computer science, and statistics. Here are a few examples:

- **Cryptography:** Elementary number theory is used in many cryptographic algorithms. For example, the RSA encryption algorithm is based on the difficulty of factoring large numbers.
- **Computer science:** Elementary number theory is used in many computer science applications, such as error-correcting codes and data compression.

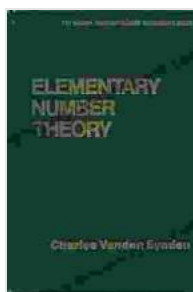
- **Statistics:** Elementary number theory is used in many statistical applications, such as hypothesis testing and confidence intervals.

Elementary number theory is a fascinating and important branch of mathematics. It has many applications in other areas of mathematics, computer science, and statistics. In this article, we have provided a comprehensive overview of elementary number theory. We have covered the basics of the subject, as well as some more advanced topics. We have also provided many examples and exercises to help you learn the material.

We encourage you to continue exploring elementary number theory. There is much more to learn about this fascinating subject.

Exercises

1. Find the GCF of 12 and 18.
2. Find the prime factorization of 100.
3. Solve the congruence $12x \equiv 18 \pmod{6}$.
4. Use elementary number theory to prove that there are infinitely many prime numbers.
5. Use elementary number theory to develop an error-correcting code.



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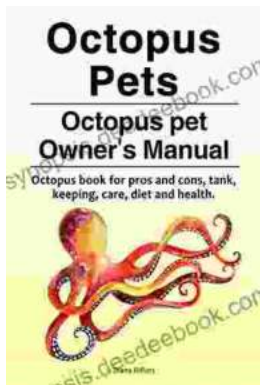
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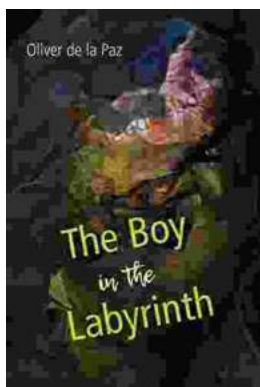
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